

Question 1.

1.a Convert 57_{10} to binary (1 Mark)

1. b Perform the following operation in binary A-B
 A= 011001 B= 010111 (3 Marks)

Show your steps clearly including the Carries/Borrows

Question 2

(Use Boolean Algebra for Question 2)

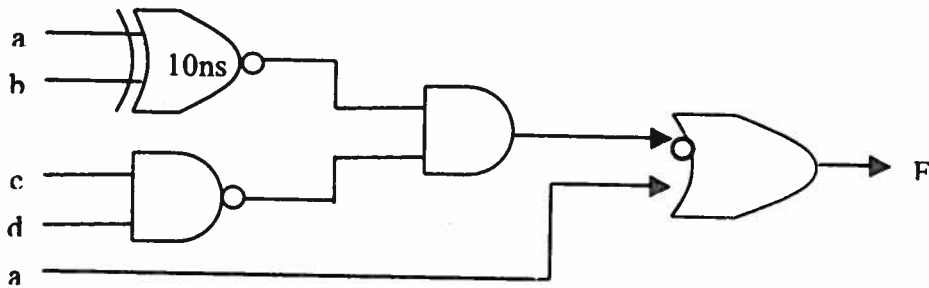
2.a Simplify to obtain minimum SOP (3 Marks)

$$F(W,X,Y,Z) = [Y \oplus (X + Y')] [(W \odot (W + Y)]$$

2.b Simplify to obtain minimum SOP (2 Marks)

$$F(a,b,c,d) = a'b'(c + d')(1 + 0) + ab(c'd + cd') + (a'b + 0)cd$$

2.c Minimize the following circuit, draw final minimized circuit. Determine speed of operation before and after minimization. All gates have equal delay of 10ns. (4 Marks)



Question 3

3.a Give minimal SOP for F(a,b,c,d) given by the following K-map (3 Marks)
 Identify the Prime Implicants and the Essential Prime Implicant clearly.

$F(a,b,c,d) =$

ab		00	01	11	10
	cd	00	01	11	10
	00	1	X	X	X
	01		1	1	1
	11	1			1
	10	X	X	X	X

3.b Give the minimal POS of $F(A,B,C) = (A + C)(B + C)$ (1 Mark)

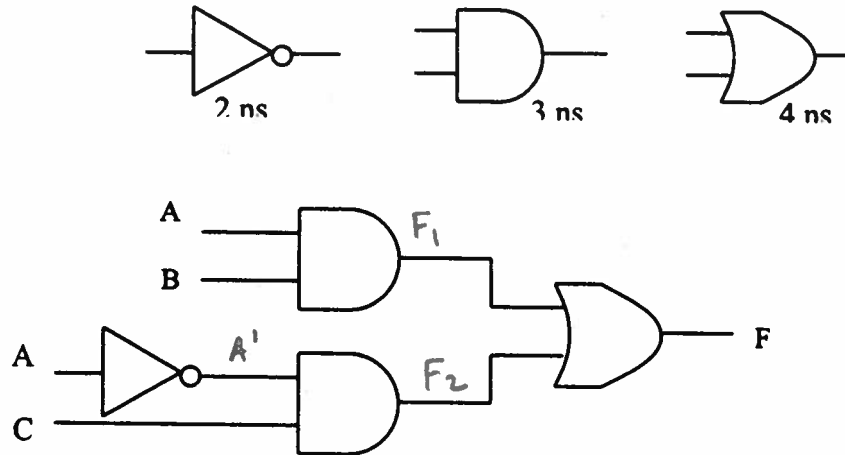
3.c Give the minterm list of $F(A,B,C,D) = A.B$ (1 Mark)

3.d Give the minimal NOR-NOR implementation of
 $F(A,B,C,D) = ABC + A(D + CB)$ (2 Marks)

Question 3

3.1 Draw the timing diagram for $F(A,B,C) = A B + (A' C)$ for the following consecutive inputs. $ABC = 000, 010, 101, 010, 111, 000$ (**follow these vectors in order given. Start from $ABC = 000$ **). Assume the following gate delays, AND = 3ns, OR gate = 4ns, and inverter = 2ns. (5 Marks)

$$F = AB + A' C$$



Question 4.

1.a Convert 57_{10} to binary

(1 Mark)

1. b Perform the following operation in binary A-B

$$A = 011001_2 \quad B = 010111_2$$

(3 Marks)

Show your steps clearly including the Carries/Borrows

Midterm Fall 2016

Q1

57 ₁₀	28	1
28	14	0
14	7	0
7	3	1
3	1	1
1	0	1

$$57_{10} = 111001_2$$

A - B

A	0	1	1	0	0	1	
B	0	1	0	1	1	1	
<hr/>							
	0	0	0	0	1	0	

Borrow ←
 Borrow ←

---	25 ₁₀
---	23 ₁₀
---	2 ₁₀

OR Using 2's Complement

$$B = 010111$$

$$101000 \text{ is Complement of } B$$

$$101001 \text{ is } 2's \text{ complement}$$

A	0	1	1	0	0	1
	1	0	1	0	0	1
<hr/>						
	0	0	0	0	1	0

← A - B a positive number

~~1101001~~

~~1101001~~ 111111
1101001

Q2

a) $A \oplus B = \bar{A}B + A\bar{B}$
 3 marks $A \odot B = AB + \bar{A}\bar{B}$

$\overline{A+B} = \bar{A}\bar{B}$
 $\overline{AB} = \bar{A} + \bar{B}$

$[Y \oplus (X+\bar{Y})]$

$Y(\overline{X+\bar{Y}}) + \bar{Y}(X+\bar{Y})$

$Y(\bar{X} \cdot Y) + \bar{Y}X + \bar{Y}$

$Y\bar{X} + \bar{Y}$

$(\bar{Y} + \bar{X})$

$[W \odot (W+Y)]$

$W(W+Y) + \bar{W}(\overline{W+Y})$

$W + \bar{W}\bar{Y}$

$(W + \bar{Y})$

$(\bar{Y} + \bar{X}) \cdot (W + \bar{Y}) = W\bar{Y} + \bar{Y} + W\bar{X} + \bar{X}\bar{Y}$
 $= \bar{Y} + W\bar{X}$

b) $F = \bar{a}b(c+d)(1+0) + ab(\bar{c}d + c\bar{d}) + (\bar{a}b + 0)cd$
 2 marks $= \bar{a}b\bar{c} + \bar{a}b\bar{d} + ab\bar{c}d + abc\bar{d} + \bar{a}bcd$ remove b
 $= \bar{a}b\bar{c} + \bar{a}b\bar{d} + ab\bar{c}d + abc\bar{d} + \bar{a}cd$
 $= \bar{d}(\bar{b}c + \bar{b}\bar{d} + cd) + ab\bar{c}d + abc\bar{d}$
 remove by consensus theorem
 $= \bar{a}\bar{b}\bar{d} + \bar{a}cd + ab\bar{c}d + abc\bar{d}$

c) from diagram directly

$F = \overline{(a \oplus b) \cdot cd} + a$
 $= (a \oplus b) + cd + a$
 $= a\bar{b} + \bar{a}b + a + cd = a + b + cd$



speed before minimization
 " after "

$3 * 10 \text{ ns} = 30 \text{ ns}$ or 33.3 MHz
 $2 * 10 \text{ ns} = 20 \text{ ns}$ or 50 MHz

Midterm 2016 Fall

Q3

a)

		ab			
		00	01	11	10
cd	00	1 ⁰	X ⁴	X ¹²	X ⁸
	01	1	1 ⁵	1 ¹³	1 ⁹
	11	1 ³	7	1 ¹⁵	1 ¹¹
	10	X ²	X ⁶	X ¹⁴	X ¹⁰

$E - PI 1 = \sum m(0, 2, 4, 6, 8, 10, 12, 14) = \bar{D}$
~~PI 2~~ $PI 2 = \sum m(8, 9, 10, 11) = A\bar{B}$
 either $PI 3 = \sum m(8, 9, 12, 13) = A\bar{C}$
 $E - PI 4 = \sum m(4, 5, 12, 13) = B\bar{C}$
 $E - PI 5 = \sum m(2, 3, 10, 11) = c\bar{B}$

$$F(A, B, C, D) = \bar{D} + \begin{matrix} A\bar{B} \\ A\bar{C} \end{matrix} + B\bar{C} + c\bar{B}$$

3 Marks

b) $F(A, B, C) = (A+C)(B+C) = (A+B)(B+C)$ 1 Mark

No minimization

c) $F(A, B, C, D) = AB$

		ab			
		00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

$$F(A, B, C, D) = \sum m(12, 13, 14, 15)$$

1 Mark

d) $F(A, B, C, D) = ABC + A(D + CB) = ABC + AD + ABC$

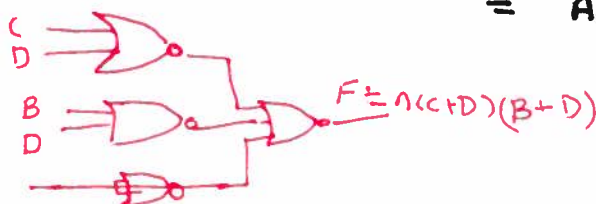
$$= ABC + AD$$

From Kmap

		AB			
		00	01	11	10
CD	00	0	0	0	0
	01	0	0	1	1
	11	0	0	1	1
	10	0	0	1	1

$$F(A, B, C, D) = A \cdot (C+D) \cdot (B+D)$$

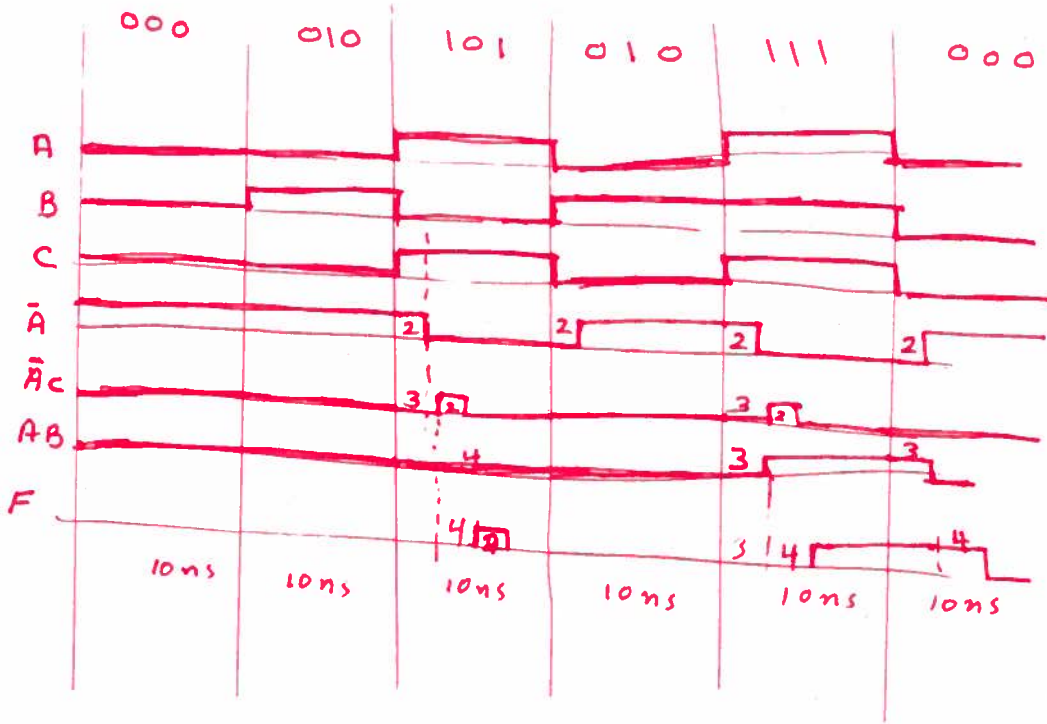
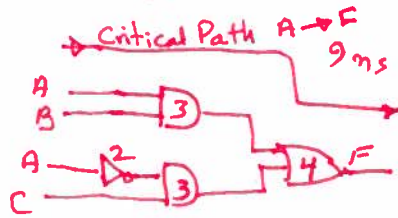
$$= \overline{\overline{A(C+D)(B+D)}} = \overline{\bar{A} + \overline{(C+D)} + \overline{(B+D)}}$$



2 Marks

Midterm Fall 2016

Q4.



5 Marks